Dynamic Reference Frames

October 2022

Topics

- Overview of Dynamic Reference Frames
- Terminology
- Rationale for Dynamic Frames
- How to Use a Dynamic Frame
- Parameterized Dynamic Reference Frames
- Defining Dynamic Reference Frames
  - Two-Vector Frames
  - "Of-Date" Frames
  - Euler Frames

  and two special cases for any of these families...
  - Frozen Dynamic Frames
  - Inertial Dynamic Frames
- Backup

What are Dynamic Frames

- Dynamic reference frames ("dynamic frames" for short) have time-dependent orientation.
  - CK- and PCK-based frames are not considered to be dynamic frames although they are time-varying.
  - Some examples are given on the next page.
- Dynamic frames are specified using a frames kernel (FK).
- The dynamic frames capability is an extension to the original SPICE frames subsystem.
- This capability enables SPICE users to conveniently use a wide variety of frames that are not "built in" to SPICE.

Examples of Dynamic Frames

- Geocentric Solar Ecliptic (GSE)
- Solar Magnetic (SM)
- Spacecraft-centered roll-celestial frame
- Geocentric Solar Magnetospheric (GSM)
- Geomagnetic (MAG)
  - Using constant north centered geomagnetic dipole
  - Using dipole direction defined by time-dependent Euler angles
- Geocentric Solar Equatorial (GSEQ)
- Solar Equatorial frame for any ephemeris object
- Orbital frame for any ephemeris object
- Earth mean equator and equinox of date
- Earth true equator and equinox of date
- Earth mean ecliptic and equinox of date
- Nadir-oriented frame for planetary orbiter
- Radial, tangential, normal (RTN)

Rationale

- **Why should SPICE offer dynamic frames implemented using a frames kernel? Instead…**
  - **a user could build a C-kernel for any frame.**
  - **SPICE could provide a limited number of "built-in" dynamic frames which wouldn't require a frames kernel.**
  - **users can create their own routines to implement dynamic frames.**
- **Benefits of using a SPICE FK for this purpose**
  - **Convenience: using a formula rather than a C-kernel avoids C-kernel creation, dissemination, storage, and consistency issues.**
  - **Flexibility: the dynamic frame mechanism enables creation of a vast variety of reference frames.**
  - **Integration: once defined, and once supporting kernels are loaded, dynamic frames may be referenced in SPICE API calls.**
  - **Correct implementation: extensive testing done by NAIF, and perhaps further verified by a user, seems less likely to result in an error.**

Using a Dynamic Frame

- Using a dynamic frame in a SPICE-based program is straightforward.
  - At program initialization:
    - » Load the needed dynamic frame kernel to make the frame definition known to SPICE.
      - Note: a single dynamic frames kernel likely contains many dynamic frame specifications.
    - » Load any kernels on which the dynamic frame depends.
      - Some dynamic frames require the use of SPK, FK, PCK, CK, or SCLK SPICE kernels.
  - Then, in calls to SPICE routines, refer to the dynamic frame by name just as you would do with other kinds of frame names, such as "J2000."

Examples of Using Dynamic Frames

- Find the 6x6 matrix to transform states from the J2000 frame to the Geocentric Solar Ecliptic (GSE) frame at the TDB epoch given by ET1.

```
CALL SXFORM( 'J2000', 'GSE', ET1, XFORM )
```

- Look up the state of Jupiter relative to the Earth in the GSE frame:

```
CALL SPKEZR( 'JUPITER', ET1,    'GSE',
             'NONE',    'EARTH', STATE, LT )
```

- You can use dynamic frames any place in SPICE where a frame name is used.
  - However, some restrictions apply to use of dynamic frames in SPICE kernels (see “Limitations” in the backup slides).

Terminology - 1

- "Frame" is an abbreviation for "reference frame."
- A frame can be defined as a set of three mutually orthogonal, unit-length vectors.
  - These vectors are called "basis vectors." The lines containing the basis vectors are the "axes" of the frame.
  - The basis vectors indicate the "positive" axis directions; we label these vectors +X, +Y, and +Z. The negatives of these vectors are labeled -X, -Y, and -Z.
  - We number the axes as follows:

    X = axis 1; Y = axis 2; Z = axis 3
- All of the frames we'll deal with are "right-handed": this means +Z is the cross product +X x +Y.
- A reference frame's orientation is defined relative to another specified frame: the "base frame."

Terminology - 2

- When we say that a frame is "time-dependent" or "time-varying," we mean:
  - The orientation of the frame is time-dependent.
  - Equivalently, the rotation between the frame and its base frame is time-dependent.
- By "evaluating" a frame or "evaluating the orientation of a frame," we mean computing the rotation between the frame and its base frame.
  - Because of their time-dependent nature, an epoch is required in order to evaluate a dynamic frame.
- In the SPICE system, frames are considered to have "centers."
  - The center of a frame is always an ephemeris object, something whose location can be specified with an SPK file.
  - Frame centers come into play when light time corrections are used: the apparent orientation of a time-dependent frame as seen by an observer is affected by the one-way light time between the frame's center and the observer.

Terminology - 3

- **When we say that a vector is "aligned" with another vector, we mean that the angular separation between the two vectors is zero.**

- **We use the terms "defining a frame" and "specifying a frame" interchangeably. Both refer to creating a frame definition in a frames kernel (FK).**

- **The notation** $$[\mathtt{theta}]_{\mathtt{n}}$$

  indicates a frame rotation of theta radians (unless other units are specified) about axis n, where n is one of {1, 2, 3}. This transformation rotates vectors by –theta radians about axis n.

Creating a Frames Kernel Defining Dynamic Frames

- To define dynamic frames using a frames kernel, a fairly detailed understanding of the SPICE dynamic frames capability is required.
- A good understanding of the basic SPICE system (in particular, the SPK and frames subsystems) is also a prerequisite.
- See the Frames Required Reading document for the most detailed documentation available.
- The rest of this tutorial is concerned with:
  - explaining the SPICE dynamic frames capability
  - showing how to create dynamic frames kernels
    - We present many dynamic frame definition examples

Types of Dynamic Frames

- NAIF envisages providing two types of dynamic frames:
  - parameterized
  - scripted

- As of now only the parametrized type is implemented within SPICE; the rest of this tutorial focuses on this type.

Parameterized Dynamic Frames

- Parameterized dynamic frames are defined using formulas.
  - The code implementing the formulas is built into SPICE.
  - The parameters used in the formulas are specified in a frames kernel.

- Parameterized dynamic frames are grouped into frame "families". Each family corresponds to a distinct, parameterized geometric formula providing a frame definition. The families are:
  - Two-Vector frames
  - Of-date frames
  - Euler frames

Defining Parameterized Dynamic Frames

- Parameterized dynamic frames are defined using "keyword=value" assignments in a frames kernel (FK).
- The following items must be specified in the frame definition:
  - Frame name
  - Frame ID code
    - The range 1400000-2000000 is reserved for people outside of the NAIF group
  - Class (= 5 for dynamic frames)
  - Class ID code (= frame ID code for dynamic frames)
  - Frame center (= name or NAIF ID code for central body)
  - Frame definition style (= 'PARAMETERIZED')
  - Base frame name (called “relative”)
    - The frame definition specifies a rotation from the dynamic frame to the base (relative) frame.
  - Frame family
  - Family-specific assignments

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Defining Parameterized Dynamic Frames

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- Rotation state
  - Possible states are 'ROTATING' and 'INERTIAL'.
    - A frame is treated as rotating or inertial for the purpose of velocity transformations.
  - The default dynamic frame rotation state is 'ROTATING'.
    - For rotating two-vector and Euler frames, the rotation state assignment can be omitted from the frame definition.
    - For "of-date" frames, the frame definition must either specify the rotation state or designate the frame as "frozen" at a specified epoch (but not both).
- Freeze epoch
  - The presence of this optional assignment in a frames kernel indicates that the frame orientation, relative to the base frame, is held constant ("frozen") at the specified epoch.
  - Most dynamic frames are not frozen.

What’s Next?

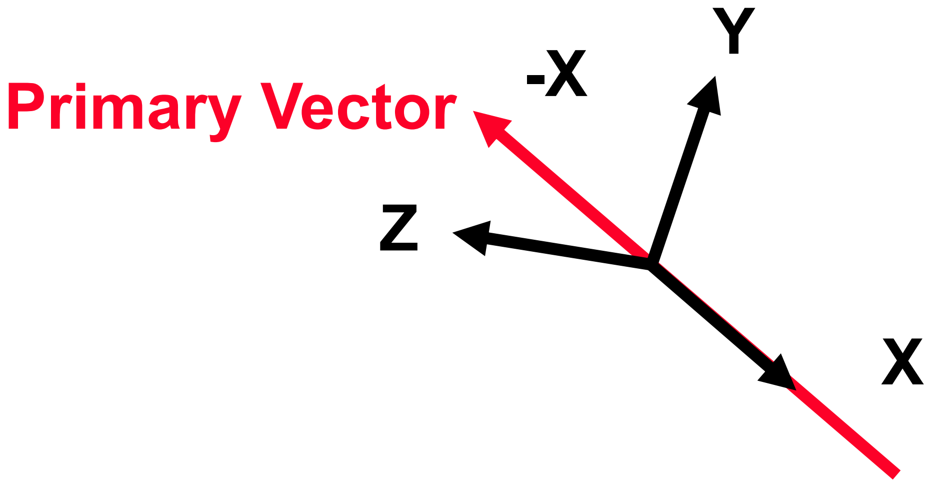
- In the next approximately 35 pages we’ll provide implementation details and examples of frames belonging to each of the parameterized dynamic frame families:
  - Two-Vector frames
  - "Of-Date" frames
  - Euler frames
  - Product frames
- We’ll also explain the optional frame attributes which can be assigned to frames belonging to the above families:
  - “Frozen”
  - “Inertial”

Two-Vector Frame Concepts - 1

- Two-vector frames are defined using two time-dependent vectors: the "primary" and "secondary" vectors.
  - Each vector may be defined by a variety of geometric means:
    - » Position vector
    - » Target near point vector
    - » Velocity vector
    - » Constant vector
- The user associates specified positive or negative axes of the two-vector frame with the primary and secondary vectors.
  - Two-vector frames are always right-handed and have orthogonal axes, so two non-parallel vectors and associations of axes with these vectors suffice to define the orientation of a frame.

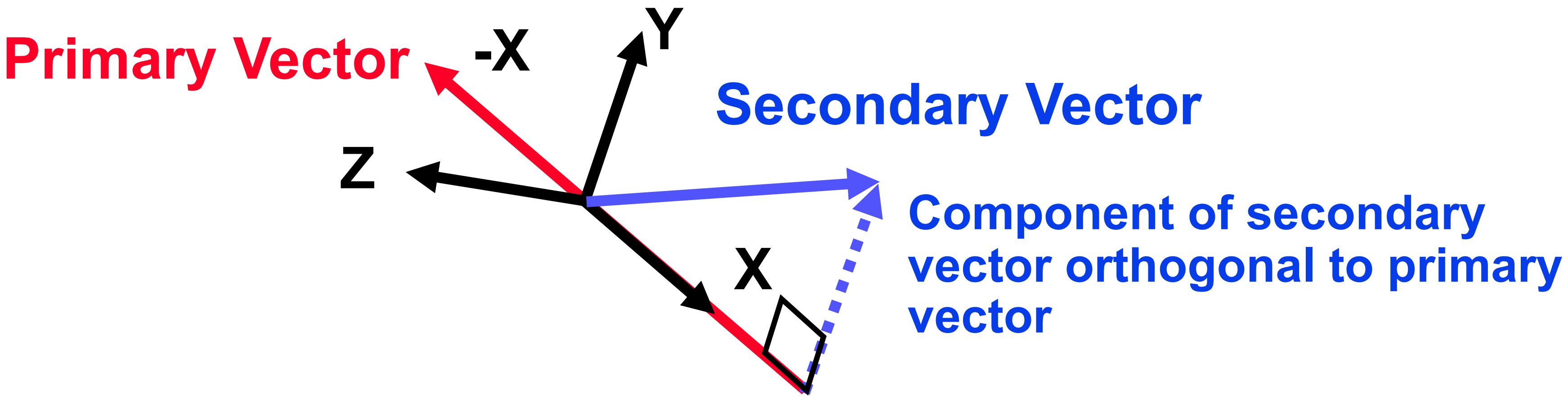
Two-Vector Frame Concepts - 2

- **Primary Vector**
  - A specified positive or negative axis of the two-vector frame is aligned with this vector.
    - The frame kernel creator assigns to this vector one of the axis designations { +X, -X, +Y, -Y, +Z, -Z }.
  - Two degrees of freedom of the frame orientation are removed by association of an axis with the primary vector. The third degree of freedom is the frame's rotation about the primary vector.
  - Example: a frame's -X axis is aligned with the primary vector:



Two-Vector Frame Concepts - 3

- **Secondary Vector**
  - A specified positive or negative axis of the two-vector frame is aligned with the component of the secondary vector orthogonal to the primary vector.
    - » The frame kernel creator associates with this vector one of the axis designations { +X, -X, +Y, -Y, +Z, -Z }, where the axis is orthogonal to that associated with the primary vector.
  - Example, continued: the frame's +Y axis is associated with the secondary vector. The component of the secondary vector orthogonal to the primary vector is aligned with the frame's +Y axis. The secondary vector thus lies in the frame's X-Y plane.



Two-Vector Frame Concepts - 4

- **Secondary Vector, continued**
  - – Typically the secondary vector itself is not orthogonal to the primary vector.
  - – However, the secondary vector must be linearly independent of the primary vector.
    - » Near-degenerate geometry can lead to extreme loss of precision.
      - This problem can be difficult to diagnose.
    - » SPICE enforces independence using a default angular separation tolerance of 1 milliradian. The angular separation of the primary and secondary vectors may not differ from 0 or Pi radians by less than this tolerance.
    - » A frame kernel creator can specify a different tolerance value. The frame kernel assignment for this is:

```
FRAME_<frame_ID>_ANGLE_SEP_TOL = <tolerance>
```

where the tolerance is given in radians.

  - – Designers of two-vector frames should ensure that the primary and secondary vectors can't become nearly parallel for any realistic evaluation epoch.

Two-Vector Frame Concepts - 5

- **In the next several pages we'll describe methods for specifying the primary and secondary vectors used in creating a dynamic frame.**

Two-Vector Frame Concepts - 6

- **Using a Position Vector**
  - Defined by the position of one ephemeris object relative to another. The frame kernel creator specifies:
    - the target
    - the observer
    - the aberration correction
      - The vector may optionally be corrected for light time and stellar aberration.
  - The epoch at which the position vector is computed is supplied via a call to a SPICE API:
    - as an input to an SPK routine, e.g. SPKEZR, SPKPOS.
    - as an input to a frame system routine, e.g. SXFORM, PXFORM.
  - The reference frame relative to which the vector is expressed is not specified by the frame kernel creator.
    - SPICE automatically selects this frame.

Two-Vector Frame Concepts - 7

- **Using a Target Near Point Vector**
  - Defined as the vector from an observer to the nearest point on a specified extended target body to that observer. The frame kernel creator specifies:
    - » the target
    - » the observer
    - » the aberration correction
      - The vector may optionally be corrected for one-way light time and stellar aberration.
      - When one-way light time correction is used, both the position and orientation of the target body are corrected for light time.
  - The extended target body is modeled as a triaxial ellipsoid.
    - » Size and shape data are given by a PCK.
  - The epoch is supplied via a SPICE API call.
  - The reference frame relative to which the vector is expressed is not specified by the frame kernel creator.
    - » SPICE automatically selects this frame.

Two-Vector Frame Concepts - 8

- Using a Velocity Vector
  - Defined by the velocity of a target ephemeris object relative to an observing ephemeris object. The frame kernel creator specifies:
    - » the target
    - » the observer
    - » the velocity reference frame
      - This frame may be distinct from the base frame.
      - Different velocity frame choices can lead to radically different two-vector frame definitions.
    - » the aberration correction
      - The velocity vector may optionally be corrected for one-way light time and stellar aberration.
      - Use of light time correction also implies evaluation of the velocity vector's frame at a light time corrected epoch: the epoch is corrected for light time between the velocity frame's center and the observer, if the velocity frame is non-inertial.
  - The epoch is supplied via a SPICE API call.

Two-Vector Frame Concepts - 9

- **Using a Constant Vector**
  - The vector is constant in a frame specified by the kernel creator.
    - » The constant vector's frame may be time-dependent.
    - » This frame may be distinct from the base frame.
  - The vector may be specified in a variety of coordinate systems.
    - » Cartesian
    - » Latitudinal
    - » Right ascension/declination (RA/DEC)
  - An observer may optionally be associated with a constant vector for the purpose of defining aberration corrections.
    - » The orientation of the constant vector's frame may optionally be corrected for one-way light time between the frame's center and the observer: if the frame is non-inertial, it is evaluated at a light time corrected epoch.

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Two-Vector Frame Concepts - 10

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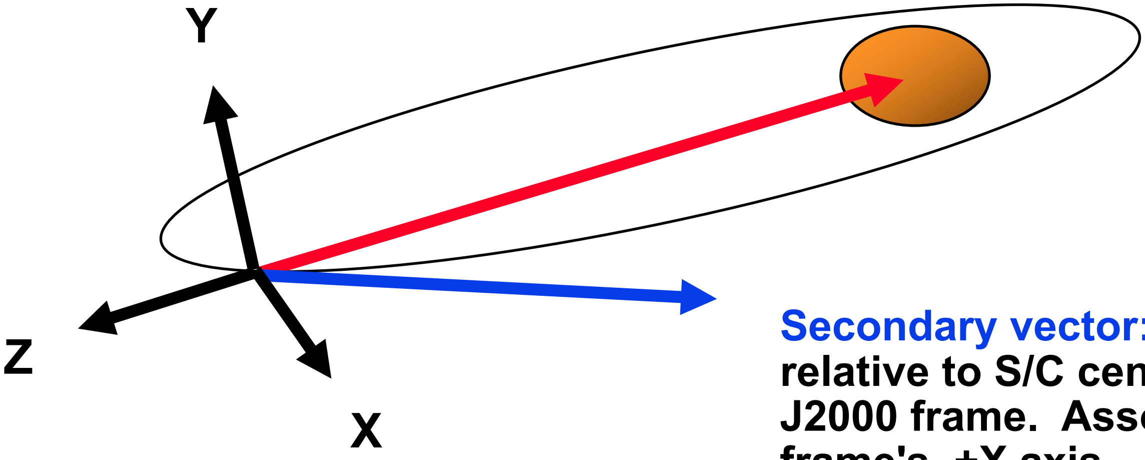
  - » A constant vector may optionally be corrected for stellar aberration due to motion of observer relative to solar system barycenter.
    - Stellar aberration can be specified without light time correction; the string indicating stellar aberration correction alone is 'S’
- – The epoch is supplied via a SPICE API call, as for position vectors.
  - » If the constant vector's frame is time-dependent, that frame is evaluated at this epoch, optionally adjusted for light time.

Two-Vector Frame Examples - 1

Nadir-Oriented Spacecraft-Centered Frame

Primary vector: spacecraft nadir direction vector. Aligned with nadir frame's -Z axis.

Y = Z x X, completing the right-handed frame.



Nadir vector can be defined to point to either:

- closest point to spacecraft on ellipsoid
- center of mass of orbited body

Secondary vector: spacecraft velocity relative to S/C center of motion in the J2000 frame. Associated with nadir frame's +X axis.

Normalized component of secondary vector orthogonal to primary vector is aligned with the nadir frame's +X axis.

See the next page for the FK specifications for this frame.

Two-Vector Frame Examples - 2

```
Nadir-Oriented Spacecraft-Centered Frame:  Frame kernel specification.

   The -Z axis points from the spacecraft toward the closest point on Mars.

   The component of the inertially referenced spacecraft velocity
   vector orthogonal to Z is aligned with the +X axis.

   The +Y axis is the cross product of the +Z axis and the +X axis.

\begindata

   FRAME_<frame_name>                = <frame_ID>
   FRAME_<frame_ID>_NAME             = <frame_name>
   FRAME_<frame_ID>_CLASS            = 5
   FRAME_<frame_ID>_CLASS_ID         = <frame_ID>
   FRAME_<frame_ID>_CENTER           = <orbiter_ID>
   FRAME_<frame_ID>_RELATIVE         = 'J2000'
   FRAME_<frame_ID>_DEF_STYLE        = 'PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY           = 'TWO-VECTOR'
   FRAME_<frame_ID>_PRI_AXIS         = '-Z'
   FRAME_<frame_ID>_PRI_VECTOR_DEF = 'TARGET_NEAR_POINT'
   FRAME_<frame_ID>_PRI_OBSERVER     = <orbiter_ID/name>
   FRAME_<frame_ID>_PRI_TARGET       = 'MARS'
   FRAME_<frame_ID>_PRI_ABCORR       = 'NONE'
   FRAME_<frame_ID>_SEC_AXIS         = 'X'
   FRAME_<frame_ID>_SEC_VECTOR_DEF = 'OBSERVER_TARGET_VELOCITY'
   FRAME_<frame_ID>_SEC_OBSERVER     = 'MARS'
   FRAME_<frame_ID>_SEC_TARGET       = <orbiter_ID/name>
   FRAME_<frame_ID>_SEC_ABCORR       = 'NONE'
   FRAME_<frame_ID>_SEC_FRAME        = 'J2000'
```

Definitions

<frame_ID> = integer frame ID code

<frame_name> = user-specified frame name

<orbiter_ID> = NAIF ID code of spacecraft

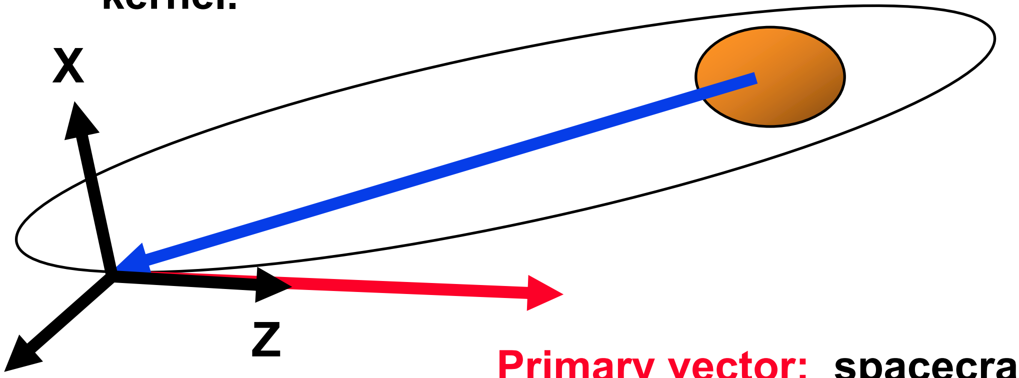
<orbiter_ID/name> = NAIF ID code or name of spacecraft

Two-Vector Frame Examples - 3

Spacecraft "View Frame"

Secondary vector: spacecraft position relative to center of motion. Associated with view frame's +Y axis in frame kernel.

X = Y x Z, completing the right-handed frame. ("Out of plane" direction)



Primary vector: spacecraft velocity relative to center of motion in J2000 frame. Aligned with view frame's +Z axis in frame kernel. ("Down track" direction)

Normalized component of secondary vector orthogonal to primary vector is aligned with the view frame's +Y axis. ("In plane" direction)

See the next page for the FK specifications for this frame.

Two-Vector Frame Examples - 4

```
Spacecraft "View Frame":  Frame kernel specification.

   The +Z axis is aligned with the J2000-referenced velocity of the
   spacecraft relative to Mars.

   The component of the spacecraft position orthogonal to +Z is aligned
   with the +Y axis.

   The +X axis is the cross product of the +Y axis and the +Z axis.

\begindata

   FRAME_<frame_name>                = <frame_ID>
   FRAME_<frame_ID>_NAME             = <frame_name>
   FRAME_<frame_ID>_CLASS            = 5
   FRAME_<frame_ID>_CLASS_ID         = <frame_ID>
   FRAME_<frame_ID>_CENTER           = <orbiter_ID>
   FRAME_<frame_ID>_RELATIVE         = 'J2000'
   FRAME_<frame_ID>_DEF_STYLE        = 'PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY           = 'TWO-VECTOR'
   FRAME_<frame_ID>_PRI_AXIS         = 'Z'
   FRAME_<frame_ID>_PRI_VECTOR_DEF = 'OBSERVER_TARGET_VELOCITY'
   FRAME_<frame_ID>_PRI_OBSERVER     = 'MARS'
   FRAME_<frame_ID>_PRI_TARGET       = <orbiter_ID/name>
   FRAME_<frame_ID>_PRI_ABCORR       = 'NONE'
   FRAME_<frame_ID>_PRI_FRAME        = 'J2000'
   FRAME_<frame_ID>_SEC_AXIS         = 'Y'
   FRAME_<frame_ID>_SEC_VECTOR_DEF = 'OBSERVER_TARGET_POSITION'
   FRAME_<frame_ID>_SEC_OBSERVER     = 'MARS'
   FRAME_<frame_ID>_SEC_TARGET       = <orbiter_ID/name>
   FRAME_<frame_ID>_SEC_ABCORR       = 'NONE'
```

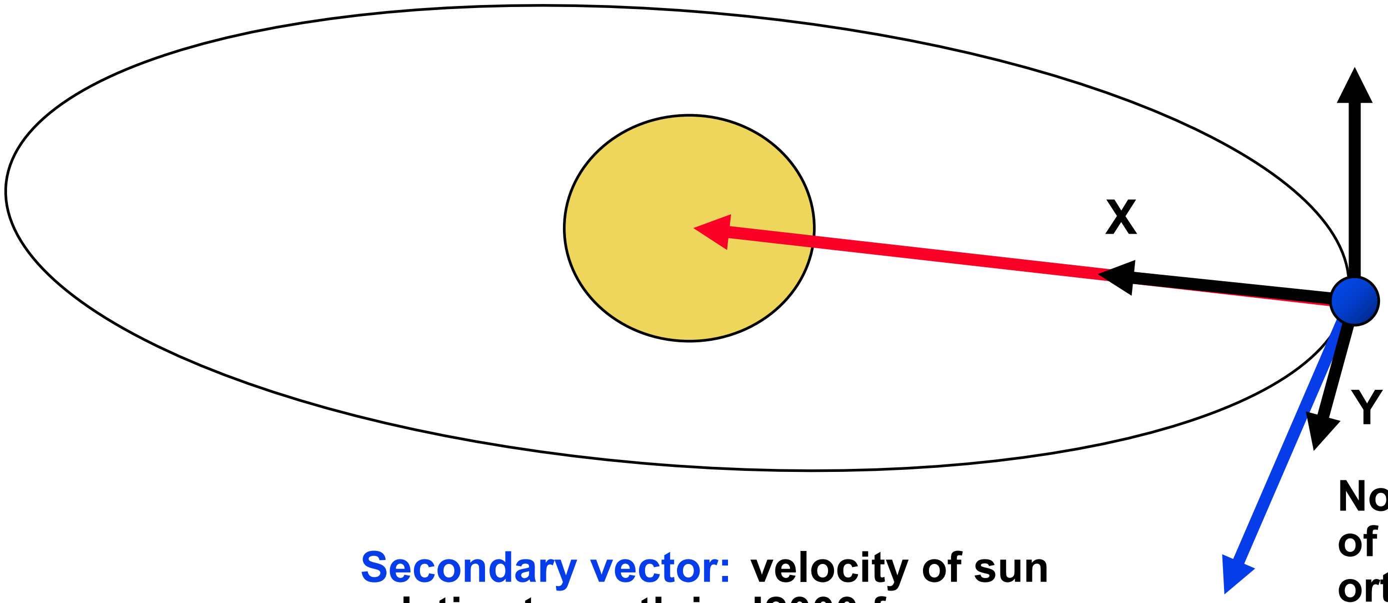
Definitions

<frame_ID>	=	integer frame ID code
<frame_name>	=	user-specified frame name
<orbiter_ID>	=	NAIF ID code of spacecraft
<orbiter_ID/name>	=	NAIF ID code or name of spacecraft

Two-Vector Frame Examples - 5

Geocentric Solar Ecliptic Frame (GSE)

Primary vector: position of sun relative to earth. Aligned with GSE frame's +X axis.



Z = X x Y, completing the right-handed frame

Secondary vector: velocity of sun relative to earth in J2000 frame. Associated with GSE frame's +Y axis in frame kernel.

Normalized component of secondary vector orthogonal to primary vector is aligned with the GSE frame’s +Y axis.

See the next page for the FK specifications for this frame.

Two-Vector Frame Examples - 6

```
Geocentric Solar Ecliptic (GSE) frame:

   +X is parallel to the geometric earth-sun position vector.

   +Y axis is the normalized component of the geometric earth-sun velocity
      vector orthogonal to the GSE +X axis.

   +Z axis is parallel to the cross product of the GSE +X axis
      and the GSE +Y axis.

\begindata

   FRAME_GSE                         =  <frame_ID>
   FRAME_<frame_ID>_NAME             = 'GSE'
   FRAME_<frame_ID>_CLASS            =  5
   FRAME_<frame_ID>_CLASS_ID         =  <frame_ID>
   FRAME_<frame_ID>_CENTER           =  399
   FRAME_<frame_ID>_RELATIVE         = 'J2000'
   FRAME_<frame_ID>_DEF_STYLE        = 'PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY           = 'TWO-VECTOR'
   FRAME_<frame_ID>_PRI_AXIS         = 'X'
   FRAME_<frame_ID>_PRI_VECTOR_DEF   = 'OBSERVER_TARGET_POSITION'
   FRAME_<frame_ID>_PRI_OBSERVER     = 'EARTH'
   FRAME_<frame_ID>_PRI_TARGET       = 'SUN'
   FRAME_<frame_ID>_PRI_ABCORR       = 'NONE'
   FRAME_<frame_ID>_SEC_AXIS         = 'Y'
   FRAME_<frame_ID>_SEC_VECTOR_DEF   = 'OBSERVER_TARGET_VELOCITY'
   FRAME_<frame_ID>_SEC_OBSERVER     = 'EARTH'
   FRAME_<frame_ID>_SEC_TARGET       = 'SUN'
   FRAME_<frame_ID>_SEC_ABCORR       = 'NONE'
   FRAME_<frame_ID>_SEC_FRAME        = 'J2000'
```

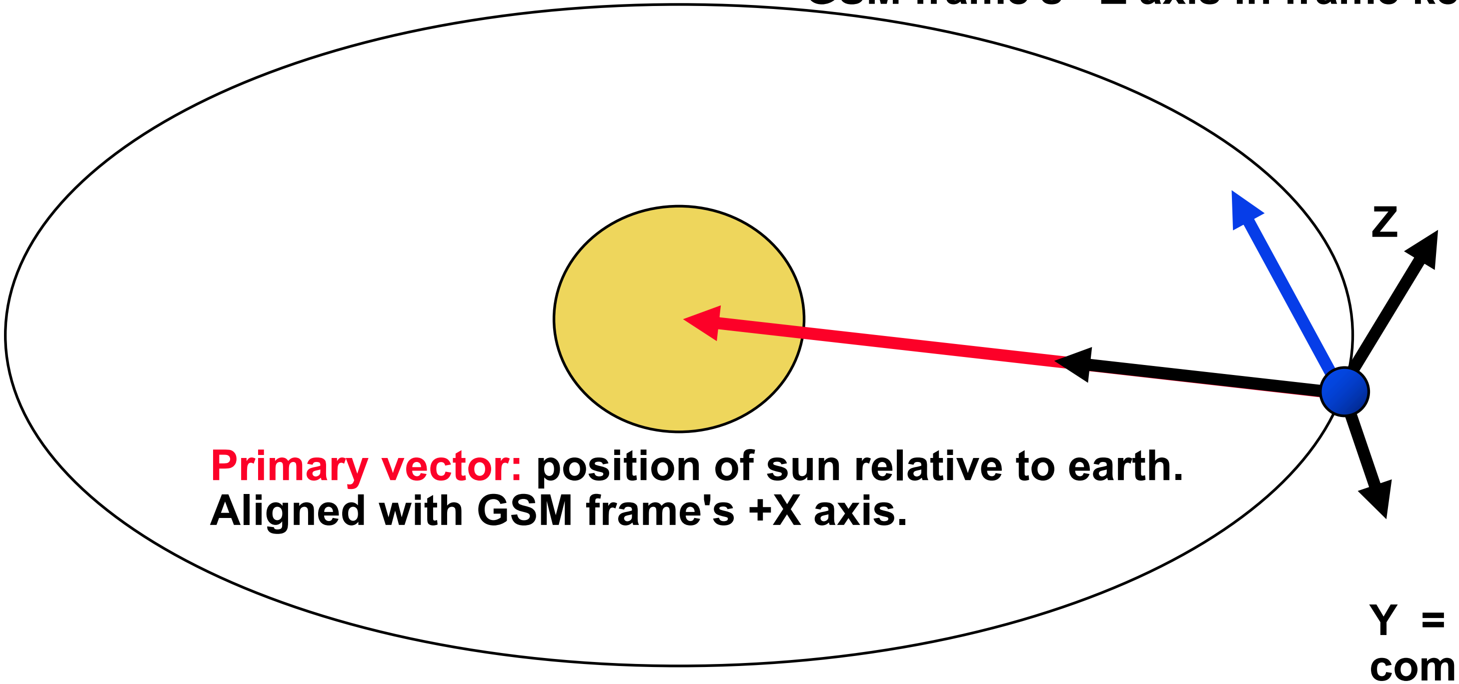
Definition

<frame_ID> = integer frame ID code

Two-Vector Frame Examples - 7

Geocentric Solar Magnetospheric Frame (GSM)

Secondary vector: North geomagnetic centered dipole in IAU_EARTH frame. Associated with GSM frame's +Z axis in frame kernel.



Normalized component of secondary vector orthogonal to primary vector is aligned with the GSM frame’s +Z axis

Primary vector: position of sun relative to earth. Aligned with GSM frame's +X axis.

Y = Z x X, completing the right-handed frame

See the next page for the FK specifications for this frame.

Two-Vector Frame Examples - 8

```
Geocentric Solar Magnetospheric (GSM) frame:

   +X is parallel to the geometric earth-sun position vector.

   +Z axis is normalized component of north centered geomagnetic dipole
      vector orthogonal to GSM +X axis.

   +Y completes the right-handed frame.

\begindata

   FRAME_GSM                      =  <frame_ID>
   FRAME_<frame_ID>_NAME          = 'GSM'
   FRAME_<frame_ID>_CLASS         =  5
   FRAME_<frame_ID>_CLASS_ID      =  <frame_ID>
   FRAME_<frame_ID>_CENTER        =  399
   FRAME_<frame_ID>_RELATIVE      = 'J2000'
   FRAME_<frame_ID>_DEF_STYLE     = 'PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY        = 'TWO-VECTOR'
   FRAME_<frame_ID>_PRI_AXIS      = 'X'
   FRAME_<frame_ID>_PRI_VECTOR_DEF = 'OBSERVER_TARGET_POSITION'
   FRAME_<frame_ID>_PRI_OBSERVER  = 'EARTH'
   FRAME_<frame_ID>_PRI_TARGET    = 'SUN'
   FRAME_<frame_ID>_PRI_ABCORR    = 'NONE'
   FRAME_<frame_ID>_SEC_AXIS      = 'Z'
   FRAME_<frame_ID>_SEC_VECTOR_DEF = 'CONSTANT'
   FRAME_<frame_ID>_SEC_FRAME     = 'IAU_EARTH'
   FRAME_<frame_ID>_SEC_SPEC      = 'LATITUDINAL'
   FRAME_<frame_ID>_SEC_UNITS     = 'DEGREES'
   FRAME_<frame_ID>_SEC_LONGITUDE =  288.43
   FRAME_<frame_ID>_SEC_LATITUDE  =   79.54
```

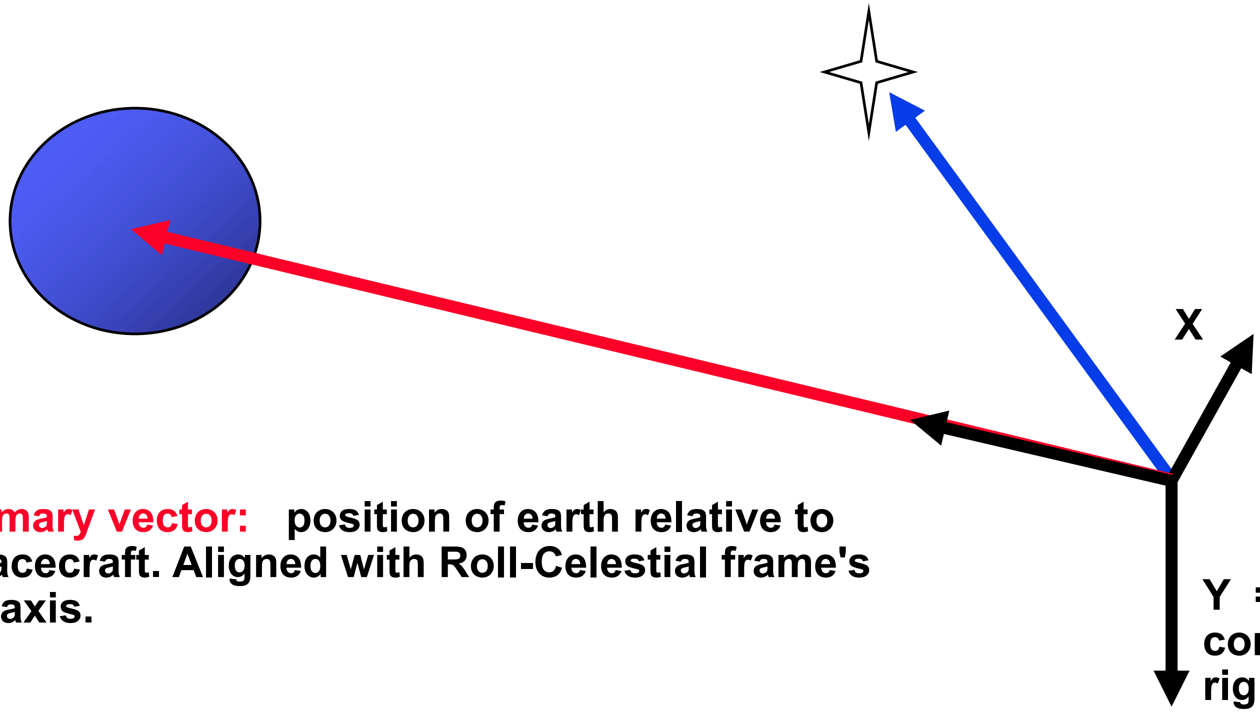
Definition

<frame_ID> = integer frame ID code

Two-Vector Frame Examples - 9

Spacecraft-Centered Roll-Celestial Frame

Secondary vector: Lock star direction in J2000 frame, corrected for stellar aberration due to spacecraft motion. Associated with Roll-Celestial frame's +X axis in frame kernel.



Normalized component of secondary vector orthogonal to primary vector is aligned with the SCR frame's +X axis

Primary vector: position of earth relative to spacecraft. Aligned with Roll-Celestial frame's +Z axis.

Y = Z x X, completing the right-handed frame

See the next page for the FK specifications for this frame.

Two-Vector Frame Examples - 10

```
 Spacecraft-centered roll-celestial frame:

   +Z is parallel to the geometric earth-sun position vector.

   +X axis is normalized component of star direction orthogonal to Z axis. The star
      direction is corrected for stellar aberration due to motion of the spacecraft.

   +Y completes the right-handed frame.

\begindata
   FRAME_<frame_name>            =  <frame_ID>
   FRAME_<frame_ID>_NAME         =  <frame_name>
   FRAME_<frame_ID>_CLASS        =  5
   FRAME_<frame_ID>_CLASS_ID     =  <frame_ID>
   FRAME_<frame_ID>_CENTER       =  <spacecraft_ID>
   FRAME_<frame_ID>_RELATIVE     = 'J2000'
   FRAME_<frame_ID>_DEF_STYLE    = 'PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY       = 'TWO-VECTOR'
   FRAME_<frame_ID>_PRI_AXIS     = 'Z'
   FRAME_<frame_ID>_PRI_VECTOR_DEF = 'OBSERVER_TARGET_POSITION'
   FRAME_<frame_ID>_PRI_OBSERVER = <spacecraft_ID/name>
   FRAME_<frame_ID>_PRI_TARGET   = 'EARTH'
   FRAME_<frame_ID>_PRI_ABCORR   = 'NONE'
   FRAME_<frame_ID>_SEC_AXIS     = 'X'
   FRAME_<frame_ID>_SEC_VECTOR_DEF = 'CONSTANT'
   FRAME_<frame_ID>_SEC_FRAME    = 'J2000'
   FRAME_<frame_ID>_SEC_SPEC     = 'RA/DEC'
   FRAME_<frame_ID>_SEC_UNITS    = 'DEGREES'
   FRAME_<frame_ID>_SEC_RA       = <star right ascension in degrees>
   FRAME_<frame_ID>_SEC_DEC      = <star declination in degrees>
   FRAME_<frame_ID>_SEC_OBSERVER = <spacecraft_ID/name>
   FRAME_<frame_ID>_SEC_ABCORR   = 'S'
```

Definitions

<frame_ID>	=	integer frame ID code
<frame_name>	=	user-specified frame name
<spacecraft_ID>	=	NAIF ID code of spacecraft
<spacecraft_ID/name>	=	NAIF ID code or name of spacecraft

"Of-Date" Frames - 1

- Of-date frames are associated with user-specified bodies and are based on user-selected dynamical models.
  - Implementations of these models are built into SPICE.
- The currently supported "of-date" frame families are
  - Mean Equator and Equinox of Date
  - True Equator and Equinox of Date
  - Mean Ecliptic and Equinox of Date
- Currently the Earth is the only supported body for of-date dynamic frames.

"Of-Date" Frames - 2

- The supported types of models are:
  - Precession
  - Nutation
  - Mean obliquity

- The of-date frame implementation is intended to be flexible…
  - The set of supported bodies can grow over time.
  - The set of supported models can grow over time.
    - » SPICE is not forever locked into using a single hard-coded implementation, such as the 1976 IAU precession model.

"Of-Date" Frames - 3

- Mean Equator and Equinox of Date Family
  - For all reference frames in this family…
    - » The frame's relationship to the J2000 frame is given by a precession model.
    - » The frame kernel creator selects a precession model from those built into the SPICE software.
      - Currently supported only for the Earth, and only the 1976 IAU precession model (Lieske model)
    - » The frame kernel creator must either specify the frame's rotation state or must designate the frame "frozen" at a specified "freeze epoch” (but not both).

"Of-Date" Frames - 4

```
 Earth mean equator and equinox of date frame:

      +Z axis is perpendicular to the mean equator of date and points north.

      +X axis is parallel to the cross product of the +Z axis and
         the north-pointing vector normal to the mean ecliptic of date.

      +Y axis completes the right-handed frame.

\begindata

   FRAME_<frame_name>                = <frame_ID>
   FRAME_<frame_ID>_NAME             = <frame_name>
   FRAME_<frame_ID>_CLASS            = 5
   FRAME_<frame_ID>_CLASS_ID         = <frame_ID>
   FRAME_<frame_ID>_CENTER           = 399
   FRAME_<frame_ID>_RELATIVE         ='J2000'
   FRAME_<frame_ID>_DEF_STYLE        ='PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY           ='MEAN_EQUATOR_AND_EQUINOX_OF_DATE'
   FRAME_<frame_ID>_PREC_MODEL       ='EARTH_IAU_1976'
   FRAME_<frame_ID>_ROTATION_STATE   ='ROTATING'
```

Definitions

<frame_ID>	=	integer frame ID code
<frame_name>	=	user-specified frame name

This is currently the only allowed value for precession model.

"Of-Date" Frames - 5

- True Equator and Equinox of Date Family
  - For all reference frames in this family…
    - » The frame's relationship to the J2000 frame is given by a precession model and a nutation model.
    - » The frame kernel creator selects models from those built into the SPICE software.
      - Currently supported only for the Earth
      - Currently only the 1976 IAU precession model (Lieske model) is allowed.
      - Currently only the 1980 IAU nutation model is allowed.
    - » The frame kernel creator must either specify the frame's rotation state or must designate the frame "frozen" at a specified "freeze epoch” (but not both).

"Of-Date" Frames - 6

```
 Earth true equator and equinox of date frame:

      +Z axis is perpendicular to the true equator of date and points north.

      +X axis is parallel to the cross product of the +Z axis and
         the north-pointing vector normal to the mean ecliptic of date.

      +Y axis completes the right-handed frame.

\begindata

   FRAME_<frame_name>                = <frame_ID>
   FRAME_<frame_ID>_NAME             = <frame_name>
   FRAME_<frame_ID>_CLASS            = 5
   FRAME_<frame_ID>_CLASS_ID         = <frame_ID>
   FRAME_<frame_ID>_CENTER           = 399
   FRAME_<frame_ID>_RELATIVE         ='J2000'
   FRAME_<frame_ID>_DEF_STYLE        ='PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY           ='TRUE_EQUATOR_AND_EQUINOX_OF_DATE'
   FRAME_<frame_ID>_PREC_MODEL       ='EARTH_IAU_1976'
   FRAME_<frame_ID>_NUT_MODEL        ='EARTH_IAU_1980'
   FRAME_<frame_ID>_ROTATION_STATE   ='ROTATING'
```

Definitions

<frame_ID>	**=**	**integer frame ID code**
<frame_name>	**=**	**user-specified frame name**

These are currently the only allowed values for precession model and nutation model.

"Of-Date" Frames - 7

- Mean Ecliptic and Equinox of Date Family
  - For all reference frames in this family…
    - » The frame's relationship to the J2000 frame is given by a precession model and an obliquity model.
    - » The frame kernel creator selects models from those built into the SPICE software.
      - Currently supported only for the Earth.
      - Currently only the 1976 IAU precession model (Lieske model) is allowed.
      - Currently only the 1980 IAU mean obliquity model is allowed.
    - » The frame kernel creator must either specify the frame's rotation state or must designate the frame "frozen" at a specified "freeze epoch."

"Of-Date" Frames - 8

```
 Earth mean ecliptic and equinox of date frame:

      +Z axis is perpendicular to mean ecliptic of date and points toward
         ecliptic north.

      +X axis is parallel to the cross product of the north-pointing
         vector normal to mean equator of date and the +Z axis.


      +Y axis completes the right-handed frame.

\begindata

   FRAME_<frame_name>                   =  <frame_ID>
   FRAME_<frame_ID>_NAME                =  <frame_name>
   FRAME_<frame_ID>_CLASS               =  5
   FRAME_<frame_ID>_CLASS_ID            =  <frame_ID>
   FRAME_<frame_ID>_CENTER              =  399
   FRAME_<frame_ID>_RELATIVE            = 'J2000'
   FRAME_<frame_ID>_DEF_STYLE           = 'PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY              = 'MEAN_ECLIPTIC_AND_EQUINOX_OF_DATE'
   FRAME_<frame_ID>_PREC_MODEL          = 'EARTH_IAU_1976'
   FRAME_<frame_ID>_OBLIQ_MODEL         = 'EARTH_IAU_1980'
   FRAME_<frame_ID>_ROTATION_STATE      = 'ROTATING'
```

Definitions

<frame_ID> = integer frame ID code

<frame_name> = user-specified frame name

These are currently the only allowed values for precession model and obliquity model.

Euler Frames - 1

- **Euler frames are defined by a time-dependent rotation relative to a base frame.**
  - **The rotation from an Euler frame to its base frame is given by three Euler angles.**
  - **Each angle is given by a separate polynomial.**
    - **The polynomials may have different degrees.**
    - **The independent variable is a time offset, in TDB seconds, from an epoch specified by the frame kernel creator.**
    - **The units associated with the angles are specified by the frame kernel creator.**
    - **The sequence of rotation axes is specified by the frame kernel creator.**
      - **The central axis must differ from the other two.**
      - **The rotation from the Euler frame to the base frame is**

$$[\text{angle\_1}]_{\text{axis\_1}}\ [\text{angle\_2}]_{\text{axis\_2}}\ [\text{angle\_3}]_{\text{axis\_3}}$$

Euler Frames - 2

- Five examples
  - Dynamic version of earth magnetospheric frame (MAG)
    - Latitude and longitude of the north centered geomagnetic dipole are given by polynomials.
  - Spinning spacecraft frame
    - The base frame could be a:
      - Built-in inertial frame
      - C-kernel frame
      - Roll-celestial frame (using lock star)
      - Nadir frame
  - Topocentric frames for tracking stations for which crustal plate motion is modeled
    - The frame rotation keeps the frame orientation consistent with the changing station location.

continued on next page

Euler Frames - 3

- Mean or true body equator and body equinox of date frame, where the body is a planet or satellite other than the earth
  - The base frame is an IAU_<body> frame.
  - The Euler frame "removes" the body's rotation about the spin axis.
- Variation on supported "of date" frame
  - An existing supported "of date" frame is used as the base frame.
  - Perturbations to the "of date" frame are expressed using Euler angles.

Euler Frames - 4

```
As an example, we construct an Euler frame called IAU_MARS_EULER. Frame
IAU_MARS_EULER is mathematically identical to the PCK frame named IAU_MARS.
The PCK data defining the underlying IAU_MARS frame are:

   BODY499_POLE_RA  = ( 317.68143  -0.1061 0.      )
   BODY499_POLE_DEC = (  52.88650  -0.0609 0.      )
   BODY499_PM       = ( 176.630   350.89198226 0. )

Relative to the angles used to define the IAU_MARS frame, the angles for our
Euler frame definition are reversed and the signs negated. Angular units are
degrees. Rate units are degrees/second, unlike the PCK units of degrees/day.

       angle_3 is  90 + RA                      angle_1 is -90 - RA
 PCK:  angle_2 is  90 - Dec       Euler Frame:  angle_2 is -90 + Dec
       angle_1 is       PM                      angle_3 is     - PM

\begindata
   FRAME_IAU_MARS_EULER                = <frame_ID>
   FRAME_<frame_ID>_NAME               = 'IAU_MARS_EULER'
   FRAME_<frame_ID>_CLASS              = 5
   FRAME_<frame_ID>_CLASS_ID           = <frame_ID>
   FRAME_<frame_ID>_CENTER             = 499
   FRAME_<frame_ID>_RELATIVE           = 'J2000'
   FRAME_<frame_ID>_DEF_STYLE          = 'PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY             = 'EULER'
   FRAME_<frame_ID>_EPOCH              = @2000-JAN-1/12:00:00
   FRAME_<frame_ID>_AXES               = ( 3 1 3 )
   FRAME_<frame_ID>_UNITS              = 'DEGREES'
   FRAME_<frame_ID>_ANGLE_1_COEFFS = ( -47.68143 0.33621061170684714E-10 )
   FRAME_<frame_ID>_ANGLE_2_COEFFS = ( -37.1135 -0.19298045478743630E-10 )
   FRAME_<frame_ID>_ANGLE_3_COEFFS = ( -176.630 -0.40612497946759260E-02 )
```

Definition

<frame_ID>	= integer frame ID code

Product Frames - 1

- **Product frames are defined by a product of one or more frame transformations.**
  - **Each factor in the product is a transformation from one frame known to the SPICE system to another such frame. The factor transformations may be time-dependent.**
- **Product frames are a generalization of TK (class 4, constant-offset) frames.**
- **A product frame implementing the transformation from a frame BASE to a frame PRODUCT has the form**

$$T^{\mathtt{PRODUCT}}_{\mathtt{BASE}} = T^{\mathtt{TO\_1}}_{\mathtt{FROM\_1}} * T^{\mathtt{TO\_2}}_{\mathtt{FROM\_2}} \ldots * T^{\mathtt{TO\_N}}_{\mathtt{FROM\_N}}$$

Product Frames - 2

- In the implementation of the product above, the factor transformations are applied in right-to-left order.
- The ``from'' and ``to'' frames of a product frame definition may be completely arbitrary. The only restriction on these frames is that the transformation from each ``from'' frame to its corresponding ``to'' frame must be computable by SPICE at the time the product frame is used.
- Because product frames are parameterized dynamic frames, limits on recursion depth for dynamic frames imply that while the factors may be dynamic frames, they may not be dynamic frames that require a level of recursion in order to evaluate their orientation.

Product Frames - 3

- A product frame is specified in a frame kernel using assignments of the following form. The "from" and "to" frames must be specified by name:

```
FRAME_<frame_name>            =  <frame_ID>
FRAME_<frame_ID>_NAME         =  <frame_name>
FRAME_<frame_ID>_CLASS        =  5
FRAME_<frame_ID>_CLASS_ID     =  <frame_ID>
FRAME_<frame_ID>_CENTER       =  <center ID or name>
FRAME_<frame_ID>_RELATIVE     = '<frame_name>'

FRAME_<frame_ID>_FAMILY       = 'PRODUCT'

FRAME_<frame_ID>_FROM_FRAMES = (  '<from_frame_1>'
                                  '<from_frame_2>'
                                      …
                                  '<from_frame_N>' )

FRAME_<frame_ID>_TO_FRAMES   = (  '<to_frame_1>'
                                  '<to_frame_2>'
                                      …
                                  '<to_frame_N>' )
```

Frozen Dynamic Frames - 1

- **A frozen dynamic frame is a "Snapshot" of a dynamic frame at a specified epoch.**
  - **The frame is frozen relative to the base frame specified by the frame kernel creator in the frame kernel definition.**
  - **The rotation from the frozen frame to the base frame is constant.**
  - **The rotation is not frozen with respect to inertial frames unless the base frame is inertial.**
  - **A frame is designated frozen by the presence of a "freeze epoch" specification in the frame definition, for example:**

```
FRAME_<FRAME_ID>_FREEZE_EPOCH = @1949-DEC-31/22:09:46.861901
```

  - **The freeze epoch is specified using SPICE text kernel rules.**
    - **The "@" syntax is used.**
    - **The time system is assumed to be TDB.**

Frozen Dynamic Frames - 2

```
 Frozen version of Earth mean equator and equinox of date frame:

      +Z axis is perpendicular to the mean equator of date and points north.

      +X axis is parallel to the cross product of the +Z axis and the
         vector normal to the mean ecliptic of date.

      +Y axis completes the right-handed frame.

\begindata

   FRAME_<frame_name>                 =  <frame_ID>
   FRAME_<frame_ID>_NAME              =  <frame_name>
   FRAME_<frame_ID>_CLASS             =  5
   FRAME_<frame_ID>_CLASS_ID          =  <frame_ID>
   FRAME_<frame_ID>_CENTER            =  399
   FRAME_<frame_ID>_RELATIVE          = 'J2000'
   FRAME_<frame_ID>_DEF_STYLE         = 'PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY            = 'MEAN_EQUATOR_AND_EQUINOX_OF_DATE'
   FRAME_<frame_ID>_PREC_MODEL        = 'EARTH_IAU_1976'
   FRAME_<frame_ID>_FREEZE_EPOCH      = @1949-DEC-31/22:09:46.861901
```

Definitions

<frame_ID> = integer frame ID code

<frame_name> = user-specified frame name

Inertial Dynamic Frames - 1

- **Inertial dynamic frames are specified by setting the rotation state to 'INERTIAL' in the rotation state assignment:**

  FRAME_<FRAME_ID>_ROTATION_STATE = 'INERTIAL'

  - **The 'INERTIAL' state implies the frame is treated as inertial for the purpose of velocity transformations.**
  - **The state transformation between any inertial frame and "inertial dynamic frame" has a zero derivative block: the state transformation matrix has the form**

```
 R(t)   |   0
--------|------
  0     |  R(t)
```

Derivative block

where R(t) is a time-dependent rotation.

continued on next page

Inertial Dynamic Frames - 2

– In contrast, for any rotating frame R(t), the state transformation between any inertial frame and R(t) has a corresponding matrix of the form

```
 R(t)   |   0
-------|------
dR(t)/dt|  R(t)
```

– The inertial rotation state:

» simplifies velocity transformations – velocities are transformed by a rotation.

» may be useful for maintaining consistency with other dynamic frame implementations.

» only makes sense if the "inertial" dynamic frame actually rotates very slowly!

Inertial Dynamic Frames - 3

```
Inertial version of Earth true equator and equinox of date frame:

     +Z axis is perpendicular to the true equator of date and points north.

     +X axis is parallel to the cross product of the +Z axis and the
        vector normal to the mean ecliptic of date.

     +Y axis completes the right-handed frame.

\begindata

   FRAME_<frame_name>                 =  <frame_ID>
   FRAME_<frame_ID>_NAME              =  <frame_name>
   FRAME_<frame_ID>_CLASS             =  5
   FRAME_<frame_ID>_CLASS_ID          =  <frame_ID>
   FRAME_<frame_ID>_CENTER            =  399
   FRAME_<frame_ID>_RELATIVE          = 'J2000'
   FRAME_<frame_ID>_DEF_STYLE         = 'PARAMETERIZED'
   FRAME_<frame_ID>_FAMILY            = 'TRUE_EQUATOR_AND_EQUINOX_OF_DATE'
   FRAME_<frame_ID>_PREC_MODEL        = 'EARTH_IAU_1976'
   FRAME_<frame_ID>_NUT_MODEL         = 'EARTH_IAU_1980'
   FRAME_<frame_ID>_ROTATION_STATE    = 'INERTIAL'
```

Definitions

<frame_ID>	=	integer frame ID code
<frame_name>	=	user-specified frame name

These are currently the only allowed values for precession model and nutation model.

Backup

- Numerical Issues
- Limitations

Numerical Issues - 1

- Two-vector frame derivatives may be inaccurate. Let R(t) represent a time-dependent rotation:
  - If R(t) depends on CK data, dR(t)/dt may be inaccurate because CK rates frequently have low accuracy.
  - If R(t) depends on velocity vectors, then dR(t)/dt depends on acceleration determined via numerical differentiation. Typically such derivatives suffer loss of accuracy.
    - However, if velocities are "well-behaved," numerically derived acceleration can be quite good. Example: GSE frame.
  - If R(t) depends on position vectors, the velocities associated with those vectors by the SPK system may not be mathematically consistent with the positions. This can happen for SPK types with separate polynomials for position and velocity, such as types 3, 8, 9, and 14.
  - If R(t) depends on aberration-corrected vectors, the associated velocities may be inaccurate due to accuracy limitations of the aberration corrections applied to velocities by the SPK system.

Numerical Issues - 2

- **Recommendations**
  - Avoid using aberration corrections in two-vector frame definitions if accurate velocity transformations are required.
  - Be aware of the accuracy of the data on which two-vector frames are based.

Limitations - 1

- **Simulated recursion:**
  - **ANSI Fortran 77 doesn't support recursion, so the SPICE dynamic frame system implements limited, simulated recursion.**
    - **» Two levels of recursion are supported for selected SPK and Frames System routines.**
  - **Users must avoid requesting "deeper" recursion than the SPICE dynamic frame system can support.**
    - **» When defining dynamic frames:**
      - **Choose J2000 as the base frame for two-vector frames.**
      - **Except for Euler frames, avoid using dynamic frames as base frames.**
      - **Try to avoid choosing a dynamic frame as the frame associated with a velocity or constant vector.**
    - **» In SPK, CK, or PCK kernels, don't use two-vector frames as the base frame relative to which ephemeris or attitude data are specified.**
      - **"Of-date" or Euler frames are OK for this purpose.**

Limitations - 2

- **Run-time efficiency:**
  - Dynamic frame evaluation typically requires more computation than is needed for CK- or PCK-based frames.
    - » For example, evaluation of a two-vector frame may involve several SPK calls.
    - » Euler frames are an exception: these are fairly efficient as long as they don't have a base frame that requires a lot of computation to evaluate.
  - To minimize the performance penalty:
    - » use J2000 as the base frame for two-vector frames.
    - » use the simplest frames possible for association with velocity or constant vectors in two-vector frame definitions.
      - Prefer non-dynamic frames to dynamic frames and inertial frames to non-inertial frames where there is a choice.